**TOPIC: INVERSE Z TRANSFORM**

**Definition:** If then is called as inverse Z-transform.

Table of Inverse Z-transform:

|  |  |  |  |
| --- | --- | --- | --- |
| Sr. No | F(z) |  | |
|  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  | - |
| 8 |  |  | - |
| 9 |  |  | - |
| 10 |  |  | - |

Methods for finding Inverse Z Transform:

1. Partial Fraction Method:
2. Power Series Method
3. Inversion Integral Method by using Residues:

Partial Fraction Method:

Examples:

1. Find

Solution: Let use partial fraction

Take inverse Z transform on both sides

…(1)

Given that

If then and

If then

Here if then

And if then

Equation (1) becomes

.

1. Find

Solution: Let use partial fraction

By putting z=2 and z=3, A=-2, B=3 OR

Take inverse Z transform on both sides

…(1)

Given that obviosely

If then

Here if then

And if then

Equation (1) becomes

.

1. Find

Solution: Let use partial fraction

By putting z=2 and z=3, A=-2, B=3

Take inverse Z transform on both sides

…(1)

Given that obviosely

If then

Here if then

And if then

Equation (1) becomes

.

1. Find

Solution: Let use partial fraction

By putting we get

Take inverse Z transform on both sides

…(1)

Given that

If then

If then

Here if then

And if | then

Equation (1) becomes

.

1. Find

Solution: Let use partial fraction

By putting we get

Take inverse Z transform on both sides

…(1)

Given that obviously

If then

Here if then

And if then

Equation (1) becomes

.

1. Find

Solution: Let

Partial fraction can only be done if degree of numerator is strictly less than the degree of denominator, so here make adjustment of z like

By putting we get

Take inverse Z transform on both sides

…(1)

Now

Therefore

If then

Here if then

And if then

Equation (1) becomes

…(2)

Now,

And

Equation (2) becomes

…

.

Alternate Method: we can write given

Now take inverse Z transform of F(z)

.

1. Find

Solution: Let

Partial fraction can only be done if degree of numerator is strictly less than the degree of denominator, so here make adjustment of z like

By putting we get

Take inverse Z transform on both sides

… (1)

Therefore

If then

Here if then

And if then

Equation (1) becomes

Inversion Integral Method by using residues:

Basic Definitions:

Zeros: A zero of an analytic function is that the value of z for which

Singular Point: A singular point of a function is the point at which the function ceases to be analytic.

Pole: Pole of F(z) is the value of z for which F(Z) is infinite.

E.g. If then

Hence, is a pole of order one or called as simple pole.

And is a pole of order 2 or called as double pole.

Residue: Residue r of at pole of order n is defined as

1. Residue r of at pole of order n=1 is defined as

1. Residue r of at pole of order n=2 is defined as

Cauchy’s Residue Theorem: If is analytic within and on closed curve C, except at a finite number of isolated singular points within C, then

Working Rule for using Inversion Integral Method:

Step 1: Find poles of by solving denominator.

Step 2: Find the expression.

Step 3: Find the residues of at all poles.

Step 4: Take sum of all residues.

Examples:

1. Using Inversion integral method find

Solution: Let

First find poles by solving denominator

are the poles of

and

Consider and multiply both sides by

Let = Residue of at pole of order n=1 is defined as

Let = Residue of at pole of order n=2 is defined as

By residue theorem,

1. Using Inversion integral method find.

Solution: Let

First find poles by solving denominator

are the poles of

and

Consider and multiply both sides by

Let = Residue of at pole of order n=1 is defined as

Let = Residue of at pole of order n=1 is defined as

By residue theorem,

1. Using Inversion integral method find

Solution: Let

First find poles by solving denominator

are the poles of

and

Consider and multiply both sides by

Let = Residue of at pole of order n=1 is defined as

Let = Residue of at pole of order n=2 is defined as

By residue theorem,

1. Using Inversion integral method find

Solution: Let

First find poles by solving denominator

are the poles of

and

Consider and multiply both sides by

Let = Residue of at pole i of order n=1 is defined as

Let = Residue of at pole of order n=1 is defined as

By residue theorem,

Applications to Solution of Difference Equation:

If then

Working rules to solve difference equation:

Step 1: Take Z transform on both sides of given equation.

Step2: Put given conditions.

Step 3: Collect the terms containing F(z) to L. H. S. and remaining R. H. S.

Step 4: Simplify F(z) by dividing coefficient of F(z).

Step 5: Express R. H. S. in terms of standard Z transform.

Step 6: Take inverse Z transform of both sides and gives f(k).

Examples:

1. Find f(k) given that

Solution: Consider given equation

Take Z transform on both sides

Put given values

Now use partial fraction

Put

Put

Take inverse Z transform on both sides

…

1. Use Z transforms to solve.

Solution: Consider given equation

Take Z transform on both sides

Put given value

Take inverse Z transform on both sides

…

1. Use Z transform to solve

Solution: Take Z transform on both sides

From equation (2) and values of A, B, C, equation (1) becomes,

Take inverse Z transform

1. Obtain the output of the system, where the input is and the system is given by where

Solution: Take Z transform on both sides of

From equation (2) and values of A, B, C, equation (1) becomes

Take inverse Z transform